A system is comprised of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector (x1,x2,x3,x4,x5), where xi is equal to 1 if component is working and is equal to 0 if component is failed.

**(a) How many outcomes are in the sample space of this experiment?**

* Each component can be in one of two states: working (1) or failed (0).
* There are 5 components.
* The total number of outcomes is 2 \* 2 \* 2 \* 2 \* 2 = 2^5 = 32.
* There are 32 possible outcomes in the sample space.

**(b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.**

* **Components 1 and 2 working:** (1, 1, x, x, x) - where x can be 0 or 1.
  + (1, 1, 0, 0, 0), (1, 1, 0, 0, 1), (1, 1, 0, 1, 0), (1, 1, 0, 1, 1), (1, 1, 1, 0, 0), (1, 1, 1, 0, 1), (1, 1, 1, 1, 0), (1, 1, 1, 1, 1) - 8 outcomes
* **Components 3 and 4 working:** (x, x, 1, 1, x)
  + (0, 0, 1, 1, 0), (0, 0, 1, 1, 1), (0, 1, 1, 1, 0), (0, 1, 1, 1, 1), (1, 0, 1, 1, 0), (1, 0, 1, 1, 1), (1, 1, 1, 1, 0), (1, 1, 1, 1, 1) - 8 outcomes
* **Components 1, 3, and 5 working:** (1, x, 1, x, 1)
  + (1, 0, 1, 0, 1), (1, 0, 1, 1, 1), (1, 1, 1, 0, 1), (1, 1, 1, 1, 1) - 4 outcomes
* **Note:** We need to remove duplicates! (1, 1, 1, 1, 1) is in all three groups.
* **Outcomes in W:**
  + (1, 1, 0, 0, 0), (1, 1, 0, 0, 1), (1, 1, 0, 1, 0), (1, 1, 0, 1, 1), (1, 1, 1, 0, 0), (1, 1, 1, 0, 1), (1, 1, 1, 1, 0), (1, 1, 1, 1, 1)
  + (0, 0, 1, 1, 0), (0, 0, 1, 1, 1), (0, 1, 1, 1, 0), (0, 1, 1, 1, 1), (1, 0, 1, 1, 0), (1, 0, 1, 1, 1)
  + (1, 0, 1, 0, 1), (1, 1, 1, 0, 1)
* **Total Outcomes in W:** 8 + 6 + 2 = 16

**(c) Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A?**

* Components 4 and 5 are failed: (x, x, x, 0, 0).
* There are 2 possibilities for each of the first 3 components.
* Total outcomes in A: 2 \* 2 \* 2 = 2^3 = 8.

**(d) Write out all the outcomes in the event AW.**

* AW means both A and W must occur.
* A: (x, x, x, 0, 0)
* W: (1, 1, x, x, x) or (x, x, 1, 1, x) or (1, x, 1, x, 1)
* AW:
  + (1, 1, 0, 0, 0), (1, 1, 1, 0, 0) - from 1&2 working
  + There are no outcomes from 3,4 working since components 4 and 5 must be failed.
  + There are no outcomes from 1,3,5 working since components 4 and 5 must be failed.
* Outcomes in AW: (1, 1, 0, 0, 0), (1, 1, 1, 0, 0)
* There are 2 outcomes in AW.

A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

**(a) Give the sample space of this experiment.**

* **Insurance:** 1 (insured), 0 (uninsured)
* **Condition:** g (good), f (fair), s (serious)
* The sample space consists of all possible combinations:
  + S={(1,g), (1,f), (1,s), (0,g), (0,f), (0,s)}

**(b) Let A be the event that the patient is in serious condition. Specify the outcomes in A.**

* A = {(1, s), (0, s)}

**(c) Let B be the event that the patient is uninsured. Specify the outcomes in B.**

* B = {(0, g), (0, f), (0, s)}

**(d) Give all the outcomes in the event Bc ∪ A**

* **Bc:** The complement of B, meaning the patient is insured.
  + Bc = {(1, g), (1, f), (1, s)}
* **A:** The patient is in serious condition.
  + A = {(1, s), (0, s)}
* **Bc ∪ A:** The union of Bc and A, meaning the outcomes in either Bc or A or both.
  + Bc ∪ A = {(1, g), (1, f), (1, s), (0, s)}

Consider an experiment that consists of determining the type of job—either blue-collar or white collar—and the political affiliation—Republican, Democratic, or Independent—of the 15 members of an adult soccer team. How many outcomes are

**(a) How many outcomes are in the sample space?**

* Each team member has two independent attributes: job type (blue-collar or white-collar) and political affiliation (Republican, Democrat, or Independent).
* **Job Types:** 2 possibilities (blue-collar, white-collar)
* **Political Affiliations:** 3 possibilities (Republican, Democrat, Independent)
* **Outcomes per person:** 2 \* 3 = 6
* **Total outcomes for 15 people:** 6^15 (6 to the power of 15)

**(b) How many outcomes are in the event that at least one of the team members is a blue-collar worker?**

* It's easier to find the number of outcomes where *none* of the team members are blue-collar and subtract that from the total outcomes.
* **None blue-collar:** This means all are white-collar.
* **Outcomes per person (white-collar):** 3 (Republican, Democrat, Independent)
* **Outcomes for 15 people (all white-collar):** 3^15 (3 to the power of 15)
* **Outcomes with at least one blue-collar:** 6^15 - 3^15

**(c) How many outcomes are in the event that none of the team members considers himself or herself an Independent?**

* **No Independents:** This means each person is either Republican or Democrat.
* **Outcomes per person (no Independent):** 2 (blue-collar or white-collar) \* 2 (Republican or Democrat) = 4
* **Outcomes for 15 people (no Independents):** 4^15 (4 to the power of 15)

Suppose that A and B are mutually exclusive events for which P(A) = .3andP(B) = .5. What is the probability that

**(a) What is the probability that either A or B occurs?**

* "Either A or B" means we're looking for the probability of their union: P(A∪B).
* For mutually exclusive events, we use the formula: P(A∪B)=P(A)+P(B)
* P(A∪B)=0.3+0.5=0.8

**(b) What is the probability that A occurs but B does not?**

* Since A and B are mutually exclusive, if A occurs, B cannot occur.
* Therefore, the probability that A occurs but B does not is simply the probability of A.
* P(A∩Bc)=P(A)=0.3

**(c) What is the probability that both A and B occur?**

* Since A and B are mutually exclusive, they cannot occur at the same time.
* Therefore, the probability of both A and B occurring is 0.
* P(A∩B)=0

A retail establishment accepts either the American Express or the VISA credit card. A total of 24 per cent of its customers carry an American Express card, 61 percent carry a VISA card, and 11 per cent carry both cards. What percentage of its customers carry a credit card that the establishment will accept?

**Given:**

* P(American Express) = 24% = 0.24
* P(VISA) = 61% = 0.61
* P(American Express and VISA) = 11% = 0.11

**Solution:**

* P(American Express or VISA) = 0.24 + 0.61 - 0.11
* P(American Express or VISA) = 0.85 - 0.11
* P(American Express or VISA) = 0.74

**Converting to Percentage:**

* 0.74 = 74%

Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty per cent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing

**Given:**

* P(Neither ring nor necklace) = 60% = 0.60
* P(Ring) = 20% = 0.20
* P(Necklace) = 30% = 0.30

**(a) What is the probability that this student is wearing a ring or a necklace?**

* We need to find P(R ∪ N), the probability of the union of R and N.
* We know that P(Neither R nor N) is the complement of P(R ∪ N).
* Therefore: P(R ∪ N) = 1 - P(Neither R nor N)
* P(R ∪ N) = 1 - 0.60 = 0.40

**(b) What is the probability that this student is wearing a ring and a necklace?**

* We need to find P(R ∩ N), the probability of the intersection of R and N.
* We can use the formula for the probability of the union of two events:
  + P(R ∪ N) = P(R) + P(N) - P(R ∩ N)
* We know P(R ∪ N), P(R), and P(N), so we can solve for P(R ∩ N):
  + P(R ∩ N) = P(R) + P(N) - P(R ∪ N)
  + P(R ∩ N) = 0.20 + 0.30 - 0.40
  + P(R ∩ N) = 0.50 - 0.40
  + P(R ∩ N) = 0.10

A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.

**Given:**

* P(Cigarettes) = 28% = 0.28
* P(Cigars) = 7% = 0.07
* P(Cigarettes and Cigars) = 5% = 0.05

**Notation:**

* C = event that a male smokes cigarettes
* G = event that a male smokes cigars

**(a) What percentage of males smokes neither cigars nor cigarettes?**

* We want to find the probability of the complement of the event that a male smokes cigarettes or cigars (or both). This is P((C ∪ G)^c).
* First, let's find the probability that a male smokes cigarettes or cigars (or both), which is P(C ∪ G).
* We use the formula:
  + P(C ∪ G) = P(C) + P(G) - P(C ∩ G)
  + P(C ∪ G) = 0.28 + 0.07 - 0.05
  + P(C ∪ G) = 0.35 - 0.05
  + P(C ∪ G) = 0.30
* Now, the probability of smoking neither is the complement:
  + P((C ∪ G)^c) = 1 - P(C ∪ G)
  + P((C ∪ G)^c) = 1 - 0.30
  + P((C ∪ G)^c) = 0.70
* Converting to percentage: 0.70 = 70%

**(b) What percentage smokes cigars but not cigarettes?**

* We want to find the probability of smoking cigars and not smoking cigarettes. This is P(G ∩ C^c).
* We know that the probability of smoking cigars is the sum of the probability of smoking only cigars and the probability of smoking both cigars and cigarettes:
  + P(G) = P(G ∩ C^c) + P(G ∩ C)
* We can rearrange this to find the probability of smoking cigars but not cigarettes:
  + P(G ∩ C^c) = P(G) - P(G ∩ C)
  + P(G ∩ C^c) = 0.07 - 0.05
  + P(G ∩ C^c) = 0.02
* Converting to percentage: 0.02 = 2%

An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. The classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

Total number of students = 100 |S| = 28 |F| = 26 |G| = 16 |S ∩ F| = 12 |S ∩ G| = 4 |F ∩ G| = 6 |S ∩ F ∩ G| = 2

We can use the Principle of Inclusion-Exclusion to find the total number of students taking at least one language class: |S ∪ F ∪ G| = |S| + |F| + |G| - |S ∩ F| - |S ∩ G| - |F ∩ G| + |S ∩ F ∩ G| |S ∪ F ∪ G| = 28 + 26 + 16 - 12 - 4 - 6 + 2 |S ∪ F ∪ G| = 70 - 22 + 2 |S ∪ F ∪ G| = 48 + 2 |S ∪ F ∪ G| = 50

**(a) If a student is chosen randomly, what is the probability that he or she is not in any of the language classes?**

The number of students not in any language class is the total number of students minus the number of students taking at least one language class: Number of students not in any class = Total students - |S ∪ F ∪ G| Number of students not in any class = 100 - 50 = 50

The probability that a randomly chosen student is not in any language class is: P(not in any class) = (Number of students not in any class) / (Total number of students) P(not in any class) = 50 / 100 = 0.5

**(b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?**

Number of students taking only Spanish = |S| - |S ∩ F| - |S ∩ G| + |S ∩ F ∩ G| = 28 - 12 - 4 + 2 = 14 Number of students taking only French = |F| - |S ∩ F| - |F ∩ G| + |S ∩ F ∩ G| = 26 - 12 - 6 + 2 = 10 Number of students taking only German = |G| - |S ∩ G| - |F ∩ G| + |S ∩ F ∩ G| = 16 - 4 - 6 + 2 = 8

Number of students taking exactly one language class = 14 + 10 + 8 = 32

The probability that a randomly chosen student is taking exactly one language class is: P(exactly one class) = (Number of students taking exactly one class) / (Total number of students) P(exactly one class) = 32 / 100 = 0.32

**(c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?**

It is easier to calculate the probability that neither of the two students is taking a language class and subtract it from 1.

Probability that the first student is not taking any language class = 50/100 Given that the first student is not taking any language class, there are 99 students remaining, 49 of whom are not taking any language class. Probability that the second student is not taking any language class (given the first is not) = 49/99

Probability that neither of the two students is taking a language class = (50/100) \* (49/99) = 0.5 \* (49/99) = 49/198

The probability that at least 1 student is taking a language class is: P(at least 1 taking a class) = 1 - P(neither taking a class) P(at least 1 taking a class) = 1 - (49/198) P(at least 1 taking a class) = (198 - 49) / 198 P(at least 1 taking a class) = 149 / 198